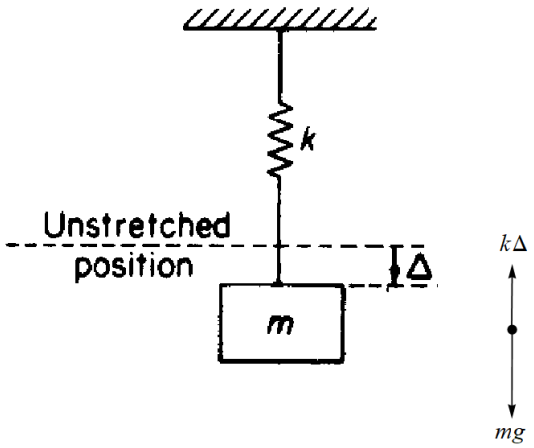
Problem 2.1

A 0.453-kg mass attached to a light spring elongates it 7.87 mm. Determine the natural frequency of the system.

Solution

Below is a schematic of a mass hanging from a spring. The free-body diagram for the mass is shown to the right.



Apply Newton's second law, taking the positive y-direction to be upward.

$$\sum F_y = k\Delta - mg = 0$$

Solve the equation for k/m.

$$\frac{k}{m} = \frac{g}{\Delta}$$

The circular frequency ω is defined by $\omega^2 = k/m$.

$$\omega^2 = \frac{g}{\Delta}$$

ω can be written in terms of the natural frequency f by $\omega = 2\pi f$.

$$(2\pi f)^2 = \frac{g}{\Delta}$$
$$2\pi f = \sqrt{\frac{g}{\Delta}}$$

Therefore, the natural frequency is

$$\begin{aligned} f &= \frac{1}{2\pi}\sqrt{\frac{g}{\Delta}} \\ &\approx \frac{1}{2\pi}\sqrt{\frac{9.81\ \frac{\text{m}}{\text{s}^2}}{.00787\ \text{m}}} \\ &\approx 5.62\ \text{Hz}. \end{aligned}$$